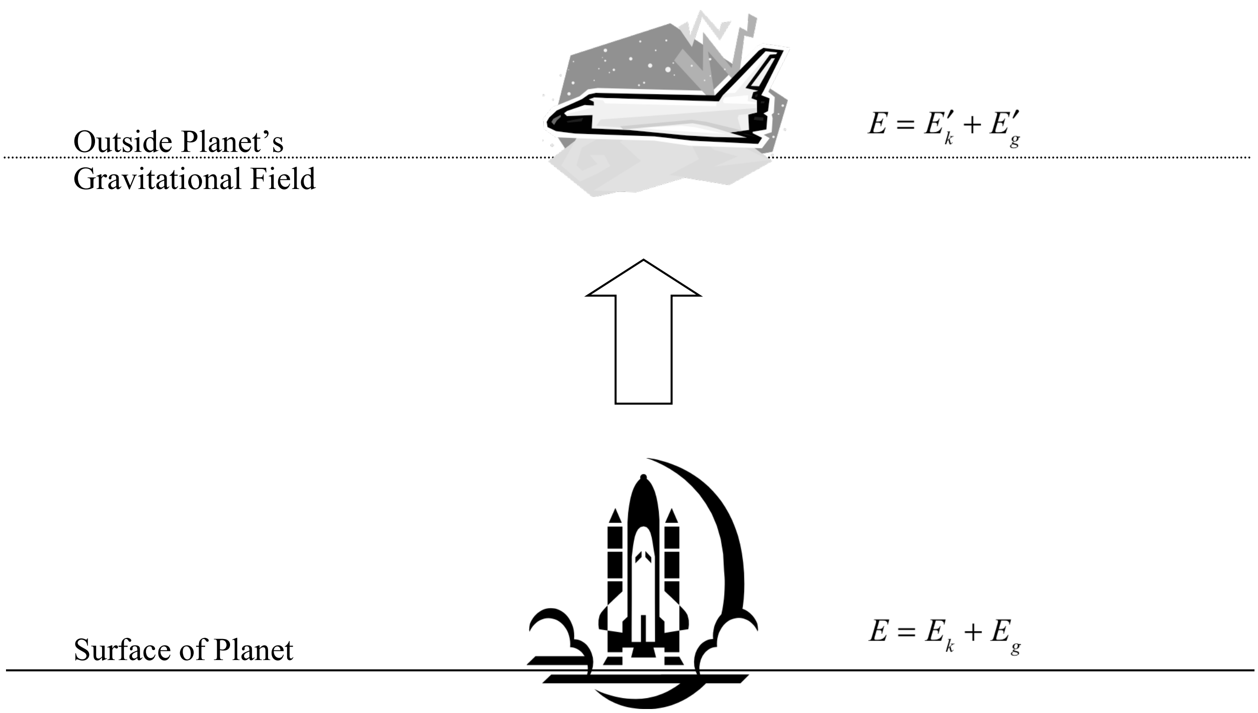
Escape Velocity

If you throw a rock straight up in the air, eventually it will come straight back down. If you fire a gun straight up in the air, the bullet will travel higher than the rock but will also eventually come straight back down. This illustrates a problem encountered by early attempts at space travel. Namely, what speed is required to escape the pull of Earth's gravity?

We can use energy analysis to answer this question. The diagram below illustrates the situation:



By Conservation of Energy:

$$E_k + E_g = E_k' + E_g'$$

We know from experience that any object which has gravitational potential energy with respect to Earth tends to fall towards the center of the planet. It stands to reason that, if we want an object to not fall towards the planet, we must move it to a location where it has zero potential energy $\left(E_g' = 0\right)$.

In addition, if we provide only the minimum amount of energy needed, then the object will come to rest at this location $\left(E_k' = 0\right)$.

Thus,

$$
\begin{aligned}
E_k + E_g &= 0 \\
\frac{1}{2}mv_e^2 - \frac{GMm}{r_p} &= 0 \\
\frac{1}{2}mv_e^2 &= \frac{GMm}{r_p} \\
\frac{1}{2}v_e^2 &= \frac{GM}{r_p} \\
v_e^2 &= \frac{2GM}{r_p}
\end{aligned}
$$

$$\boxed{v_e = \sqrt{\frac{2GM}{r_p}}}$$

Where v_e is the **minimum** escape velocity, M is the mass of the planet, and r_p is the radius of the planet.

Example 1
Find the escape velocity of a rocket on Earth.

Binding Energy

A rocket whose total energy is negative will not be able to escape from Earth's gravitational pull, and is said to be bound to Earth. The **binding energy** of any mass is the amount of additional kinetic energy it needs to escape Earth's gravity.

Binding Energy on the Surface

For a rocket of mass m, at rest on Earth's surface:

$$\begin{aligned} E &= E_k + E_g \\ &= 0 + \left(-\frac{GM_E m}{r_E}\right) \\ &= -\frac{GM_E m}{r_E} \end{aligned}$$

To escape Earth's gravity, the total energy must be reduced to zero. To do this, we must add energy equal to

$$\frac{GM_E m}{r_E}$$

Thus, the binding energy for a mass m, at rest on Earth's surface is

$$\boxed{E_B = \frac{GM_E m}{r_E}}$$

Binding Energy in Orbit

For an object in orbit, the total energy is given by:

$$\begin{aligned} E &= E_k + E_g \\ &= \frac{1}{2}mv^2 + \left(-\frac{GM_E m}{r_E}\right) \\ &= \frac{1}{2}mv^2 - \frac{GM_E m}{r_E} \end{aligned}$$

To escape Earth's gravity, the total energy must be reduced to zero. To do this, we must add energy equal to

$$\frac{GM_E m}{r_E} - \frac{1}{2}mv^2$$

Thus, the binding energy for a mass m, in orbit around Earth is

$$\boxed{E_B = \frac{GM_E m}{r_E} - \frac{1}{2}mv^2}$$

Homework
Gravitational Potential Energy Worksheet #2

Gravitational Potential Energy Worksheet #2

1. Calculate the escape velocity of the moon. ($2.4\ km/s$)

2. Calculate the escape velocity from each planet in our solar system. (see below)

3. A distant planet has a mass of $0.82M_E$ and a radius of $0.95R_E$. What is the escape speed from this planet? ($10.4\ km/s$)

4. Calculate the binding energy for a $500\ kg$ rocket at rest on the surface of each planet in our solar system. (see below)

5. With what speed would an object need to be projected from the surface of the Sun in order to escape its gravitational pull? ($616\ km/s$)

Answer to question #2 and 4

Planet	**Escape Velocity** (km/s)	**Binding Energy** (J)
Mercury	4.3	4.2×10^{9}
Venus	10.4	2.6×10^{10}
Earth	11.2	3.1×10^{10}
Mars	5.0	6.2×10^{9}
Jupiter	60.2	8.8×10^{11}
Saturn	36.0	3.1×10^{11}
Uranus	22.3	1.1×10^{11}
Neptune	24.9	1.4×10^{11}
Pluto	5.2	6.7×10^{9}